Welcome to the first edition of my AS Revision Guide http://www.mathslearn.co.uk

Contained within you will find the following topics. Each topic links to YOUTUBE videos.

There are also questions contained within and a main section of MOCK QUESTIONS at the end. I suggest you attempt these questions to help your revision become active and hence effective. Attempting questions is the best way to revise for an examination.

Solutions to the questions will be released soon along with more examples shortly. As a subscriber you get access to these more quickly.

For a free, no obligation subscription e-mail info@mathslearn.co.uk with AS MATHS in the header.

Topics:

- The equation of a line
- The equation of a circle
- Quadratic Inequalities
- Solving Cubic equations and the factor theorem
- Graph sketching
- Solving Trig Equations
- Applications of Trigonometry
- Binomial Expansion

The next edition will focus on further areas including

- Integration
- Sequences and Series
- Statistics and Probability
- Logarithms

Please feel free to share this revision guide to friends. As a subscriber, please feel free to contact me at any time with questions you may have and I will do my best to help.

The general equation of a line: (Video Link: http://youtu.be/7G8EwEc5xLw)

If we know the gradient, m, of a straight line and a coordinate (x_1, y_1) then the equation of the line can be written as:

$$y - y_1 = m(x - x_1)$$

Example: find the equation of a line with a gradient of 3 passing through (2,4)

$$y - 4 = 3(x - 2)$$

$$y - 4 = 3x - 6$$

$$y = 3x - 2$$

Key fact: lines a parallel IF they have the same gradient

Example: find the equation of the line which is parallel to $y = 4x - 1$ and which passes through $(2,5)$

Because the line is parallel, then it must have a gradient of 4

So, $y - 5 = 4(x - 2)$

$$y - 5 = 4x - 8$$

$$y = 4x - 3$$

Key fact: to find a perpendicular gradient, we need to reciprocate and change the sign

Example: find the equation of the line perpendicular to $y = -\frac{3x}{2} + 1$, passing through $(-2,5)$

The perpendicular gradient must be $+\frac{2}{3}$ so the equation of the line must be:

$$y - 5 = \frac{2}{3}(x - (-2))$$

$$y - 5 = \frac{2}{3}(x + 2)$$

$$3y - 15 = 2(x + 2)$$

$$3y - 15 = 2x + 4$$

$$3y = 2x + 19$$

$$y = \frac{2x}{3} + \frac{19}{3}$$

The equation of a perpendicular bisector

A perpendicular bisector passes through the midpoint of TWO coordinates

Find the perpendicular bisector between $(-3,2)$ and $(1,8)$

The midpoint is:

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-3+1}{2},\frac{2+8}{2}\right)=(-1,5)$$

The gradient of the line passing through the two coordinates is:

$$\frac{y_2-y_1}{x_2-x_1}=\frac{8-2}{1--3}=\frac{6}{4}=\frac{3}{2}$$

The perpendicular gradient is therefore $-\frac{2}{3}$

So the equation of the perpendicular bisector is given by:

$$y-5=-\frac{2}{3}\left(x-(-1)\right)$$

$$3y-15=-2(x+1)$$

$$3y-15=-2x-2$$

$$3y=-2x+13$$

$$y=-\frac{2x}{3}+\frac{13}{3}$$

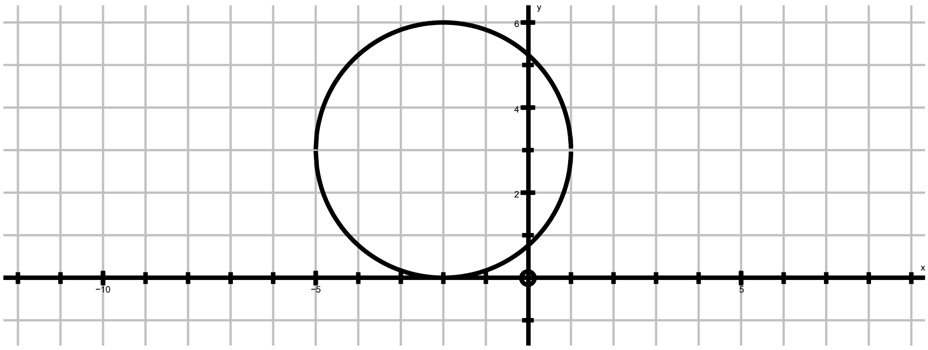
Circles

The equation of a circle is given by:

$$(x-a)^2+(y-b)^2=r^2$$

Where the centre of the circle is (a,b) and the radius is r

Example: Write down the equation of the following circle



The centre is at $(-2,3)$ and the radius is 3. So the equation of the circle is given by:

$$(x+2)^2+(y-3)^2=9$$

Example: Write down the centre and radius of the following circles

1) $(x-3)^2+(y+1)^2=25$

The centre is at $(3,-1)$ and the radius is $\sqrt{25}=5$

2) $x^2+(y-2)^2=17$

The centre is $(0,2)$ and the radius is $\sqrt{17}$. We can leave this as a surd

3) $x^2+4x+y^2-6y=2$

To work this we COMLPETE THE SQUARE (Video Link: http://youtu.be/37zUIvDMZQQ)

$$x^2+4x=(x+2)^2-2^2=(x+2)^2-4$$

$$y^2-6y=(y-3)^2-3^2=(y-3)^2-9$$

Substitute these into the original expression

$$(x+2)^2-4+(y-3)^2-9=2$$

$$(x+2)^2+(y-3)^2=2+9+4=15$$

$$(x+2)^2+(y-3)^2=15$$

The centre is $(-2,3)$ and the radius is $\sqrt{15}$. Again, we leave this as a surd.

We are often asked to solve problems involving circles and tangents

A tangent to a circle JUST touches the circle once and is PERPENDICULAR TO THE RADIUS

Example: Find the equation of the tangent at (3,1) to the circle: $(x-2)^2+(y+3)^2=17$

Step 1: Find the gradient of the radius which passes through the centre (2,3) and the point (3,1)

$$\frac{y_2-y_1}{x_2-x_1}=\frac{1-3}{3-2}=-\frac{2}{1}=-2$$

Step 2: The gradient of the TANGENT is perpendicular to the radius

$$m=-\frac{1}{-2}=\frac{1}{2}$$

Step 3: Use the general equation of the line to find the equation of the tangent

$$y-y_1=m(x-x_1)$$

$$m=\frac{1}{2}$$

$$(x_1,y_1)=(3,1)$$

$$y-1=\frac{1}{2}(x-3)$$

$$2y-2=1(x-3)$$

$$2y-2=x-3$$

$$2y=x-1$$

$$y=\frac{x}{2}-\frac{1}{2}$$

Possible extension question: Find where this tangent crosses the axes

The y-intercept is where $x=0$

So, $y=0-\frac{1}{2}=-\frac{1}{2}$

The x-intercept is where $y=0$, so $0=\frac{x}{2}-\frac{1}{2}$

$$\frac{x}{2}=\frac{1}{2}\quad so,\quad x=1$$

Quadratic inequalities (Video Link: http://youtu.be/1ECBGWtgieE)

Solve $x^2 + 5x - 14 > 0$

To solve this we consider the graph $y = x^2 + 5x - 14$

We can sketch the graph by first factorising this to get $y = (x + 7)(x - 2)$

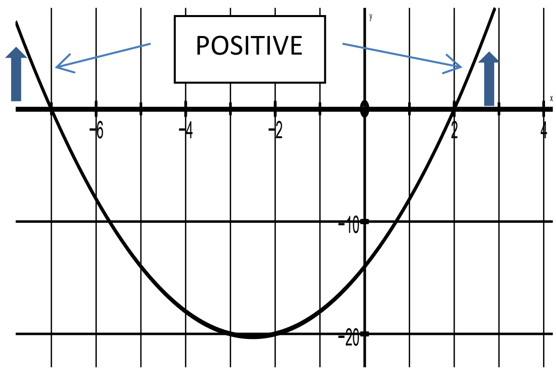
So we know the roots are at $x = -7$ and $x = 2$

If we consider the graph:

The inequality $x^2 + 5x - 15 > 0$ is asking WHEN the graph is positive

In other words, the question is asking for what values of x is the graph ABOVE the x-axis. It is clear from the sketch that this happens when:

$$x < -7 \; or \; x > 2$$



An application of quadratic inequalities (Video link: http://youtu.be/mozqJusLW9c)

Find the values of x for which $y = x^3 + 2x^2 + x + 2$ is a DECREASING FUNCTION

The function is classified as decreasing when the GRADIENT IS NEGATIVE

We find the gradient by differentiating [see later in guide], $\frac{dy}{dx} = 3x^2 + 4x + 1$

So we need to solve the inequality:

$$3x^2 + 4x + 1 < 0$$

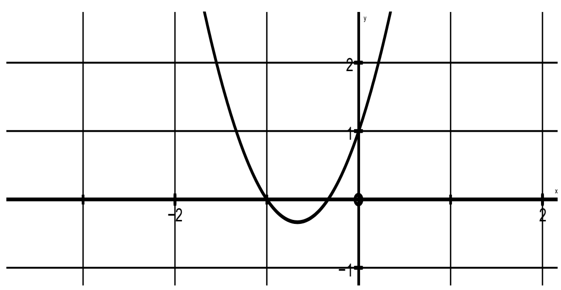
We begin by factorising to get: $(3x + 1)(x + 1) < 0$

The roots are at $x = -\frac{1}{3} \; and \; x = -1$

If we consider the Quadratic Graph with roots at the points -1 and $-\frac{1}{3}$

It is clear that the graph is below the axis when

$$-1 < x < -\frac{1}{3}$$



Solving cubic equations

Suppose we are asked to solve $f(x) = x^3 + 2x^2 - x - 2 = 0$

We first of all try to find a root of this equation by substituting in values of for x

When $x = 1$, $f(1) = (1)^3 + 2(1)^2 - 1 - 2 = 1 + 2 - 1 - 2 = 0$

This tells us that, because the remainder is zero, that $x = 1$ is a root

So, using the factor theorem, $(x - 1)$ must be a factor

We now work out $f(x) \div (x - 1)$ to work out the other factors

(See video link: http://youtu.be/Wj5PGYJxLvk)

	x^2	$3x$	$+2$
x	x^3	$+3x^2$	$+2x$
-1	$-x^2$	$-3x$	-2

We have found that: $(x^3 + 2x^2 - x - 2) \div (x - 1) = x^2 + 3x + 2$

So,

$$x^3 + 2x^2 - x - 2 = (x - 1)(x^2 + 3x + 2)$$

$$= (x - 1)(x + 2)(x + 1)$$

The three roots are therefore $x = 1, x = -2, x = -1$

Should the quadratic factor not factorise into a pair of linear brackets, then we would use the quadratic formula to find the final two solutions

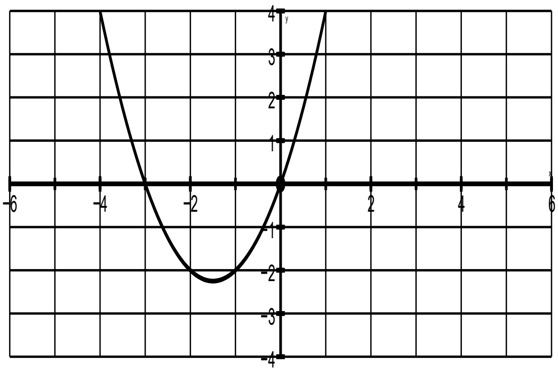
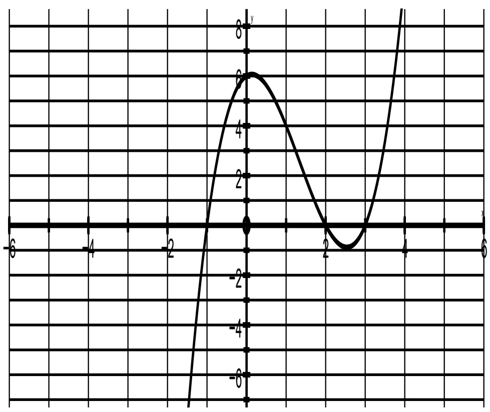
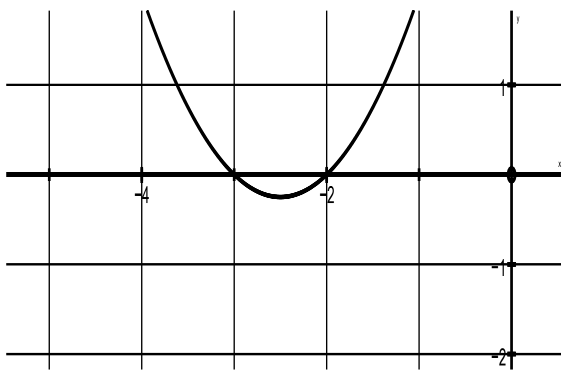
Further example 1: http://youtu.be/e3Ti05S4eXw

Further example 2: http://youtu.be/7rvKBc8wwi8

Graph Sketching is a fundamental skill in any area of Mathematics and in this edition we start to consider the KEY skills in sketching graphs. In our next edition we will extend these ideas further to even more complex polynomials (Video Link: http://youtu.be/CdeG0JM9vSg)

Success in graph sketching depends upon a number of key success criteria

1) Factorise to identify the roots (where the graph hits the $x - axis$)
2) Identify the $y - intercept$ by finding y when $x = 0$
3) Understanding the shape of the graph

$y = x^2 + 3x$ 1) $y = x^2 + 3x = x(x + 3)$ Roots at $x = 0$ and $x = -3$ 2) When $x = 0, y = 0^2 + 3(0) = 0$ Curve has a y-intercept of 0 3) The graphs is a quadratic so is a 'smiley face'	
$y = (x - 2)(x + 1)(x - 3)$ 1) Already factorised so roots are at: $x = 2, x = -1 \; and \; x = 3$ 2) When $x = 0, y = (-2)(1)(-3) = 6$ The y-intercept is 6 3) The graph is a cubic so has an 's-shape'	
$y = x^2 + 5x + 6$ 1) Factorise $y = (x + 3)(x + 2)$ So roots are at $x = -3$ and $x = -2$ 2) When $x = 0, y = 0^2 + 5(0) + 6 = 6$ y-intercept is equal to 6 3) The graph is a quadratic – so is a 'smiley face'	

Solving Basic Equations in Trigonometry

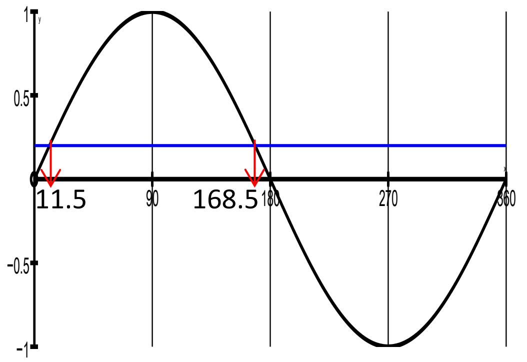
If you are asked to solve $sinx = 0.2$ then we can find ONE solution immediately by using the $\sin^{-1}$.

$$x = \sin^{-1} 0.2 = 11.5°$$

However a quick consideration of the graph shows that there are more values which satisfy the equation

If I draw the line $y = 0.2$ over $y = sinx$ then we can see that there are TWO intersections for the values of x between 0 and 360°

The first value is $x = 11.5$
The second value is at $x = 180 - 11.5 = 168.5°$



The symmetries of $cosx$ and $sinx$ can be summarised quite quickly by considering four examples

These examples show to find ALL values of x where $0 \leq x \leq 360°$

Equation	Step 1	Solution 1	Solution 2
$sinx = 0.4$	$x = \sin^{-1} 0.4$ $= 23.6°$	$23.6°$	$180 - 23.6°$ $= 156.4°$
$sinx = -0.3$	$x = \sin^{-1} -0.3$ $= -17.45°$	$180 + 17.45$ $= 197.45°$	$360 - 17.45$ $= 342.55°$
$cosx = 0.45$	$x = \cos^{-1} 0.45$ $= 63.3°$	$63.3°$	$360 - 63.3$ $= 296.7°$
$cosx = -0.7$	$x = \cos^{-1} -0.7$ $= 134.4°$	$134.4°$	$360 - 134.4$ $= 225.6°$

Further example:

$$solve: \cos^2 x = 0.64$$

Step 1: We find the square root: $cosx = \pm\sqrt{0.64} = \pm 0.8$

We then solve the two equations separately:

$cosx = 0.8$ $x = \cos^{-1} 0.8 = 36.9°$ OR $x = 360 - 36.9 = 323.1°$	$cosx = -0.8$ $x = \cos^{-1} -0.8 = 116.9°$ OR $x = 360 - 116.9 = 243.1°$

So the four solutions are: $36.9°, 116.9°, 243.1°$ *and* $323.1°$

Solving Harder Equations involving $sinx$ and $cosx$

(Video Link: http://youtu.be/nzM1lw4UI0E)

A key identity which you need to know is: $\cos^2 x + \sin^2 x = 1$

This means that: $\cos^2 x = 1 - \sin^2 x$ OR $\sin^2 x = 1 - \cos^2 x$

We can use this to solve more difficult equations:

Solve the following equation for $0 \leq x \leq 360°$:

$$9 - 3sinx = 10\cos^2 x$$

Substitute in $\cos^2 x = 1 - \sin^2 x$ to write equation in terms of $sinx$ only

$$9 - 3sinx = 10(1 - \sin^2 x)$$

For ease, let $sinx = s$, then expand and rearrange to get all terms onto one side

$$9 - 3s = 10 - 10s^2$$

$$10s^2 + 9 - 3s = 10$$

$$10s^2 - 3s - 1 = 0$$

$$(5s + 1)(2s - 1) = 0$$

Option 1:	Option 2:
$2s - 1 = 0 \Rightarrow s = \frac{1}{2}$ $sinx = \frac{1}{2}$ $x = \sin^{-1}\left(\frac{1}{2}\right) = 30°$ Or, $x = 180 - 30 = 150°$	$5s + 1 = 0 \Rightarrow s = -\frac{1}{5}$ $sinx = -\frac{1}{5}$ $x = \sin^{-1}\left(-\frac{1}{5}\right) = -11.5°$ So, $x = 180 + 11.5 = 191.5°$ Or, $x = 360 - 11.5 = 348.5°$

Differentiation (Video link: http://youtu.be/ksh7XoRrSlE)

We differentiate to find the gradient of a given curve or function. To differentiate we MULTIPLY BY THE POWER AND THEN SUBTRACT ONE FROM THE POWER On the right are some key examples of differentiation	$y = ax^n \quad \frac{dy}{dx} = anx^{n-1}$ $y = x^4 + 7x - 3 \quad \frac{dy}{dx} = 4x^3 + 7$ $y = \sqrt{x} = x^{\frac{1}{2}} \quad \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ $y = \frac{5}{x^3} + \sqrt[3]{x} = 5x^{-3} + x^{\frac{1}{3}}$ $\frac{dy}{dx} = -15x^{-4} + \frac{1}{3}x^{-\frac{2}{3}}$

APPLICATION 1: FINDING THE EQUATION OF A TANGENT OR NORMAL

(Video Link http://youtu.be/BhIKpgCbLRo)

Suppose we are asked to find the equation of the tangent to the curve $y = 3x^2 - 2x$ at $x = 1$

Step 1: Differentiate to find the gradient at $x = 1$

$\frac{dy}{dx} = 6x - 2$ at $x = 1$ the gradient is therefore $m = 6(1) - 2 = 4$

Step 2: Find the corresponding y coordinate using the original equation $y = 3x^2 - 2x$

At $x = 1$, $y = 3(1)^2 - 2(1) = 3 - 2 = 1$

Step 3: Find the equation of the tangent using $y - y_1 = m(x - x_1)$

$$y - 1 = 4(x - 1)$$

$$y = 4x - 3$$

Suppose we are asked to find the equation of the NORMAL to the curve $y = 3x^2 - 2x$ at $x = 2$

Step 1: Differentiate to find the gradient at $x = 2$

$\frac{dy}{dx} = 6x - 2$ at $x = 2$ the gradient is therefore $m = 6(2) - 2 = 10$, so gradient of normal is $-\frac{1}{10}$

Step 2: Find the corresponding y coordinate using the original equation $y = 3x^2 - 2x$

At $x = 2$, $y = 3(2)^2 - 2(2) = 12 - 4 = 8$

Step 3: Find the equation of the tangent using $y - y_1 = m(x - x_1)$

$$y - 8 = -\frac{1}{10}(x - 2) \qquad 10y - 80 = -1(x - 2) \qquad 10y - 80 = -x + 2$$

$$10y = -x + 82$$

Finding and classifying TURNING POINTS/STATIONARY POINTS (http://youtu.be/WLHAwlJzUhw)

KEY POINT: The turning point or stationary point or a graph is where the gradient is equal to ZERO

Example: Find the coordinates of the turning points of $y = x^3 + 3x^2 - 24x + 7$

Step 1: Differentiate $\frac{dy}{dx} = 3x^2 + 6x - 24$

Step 2: Solve to find where $\frac{dy}{dx} = 0$

$$3x^2 + 6x - 24 = 0 \quad \Longrightarrow \quad x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0 \quad \Longrightarrow \quad x = -4 \quad or \quad x = 2$$

Step 3: Find the y coordinates

$$x = -4 \implies y = (-4)^3 + 3(-4)^2 - 24(-4) + 7 = 87$$

$$x = 2 \implies y = (2)^3 + 3(2)^2 - 24(2) + 7 = -21$$

So the coordinates of the turning points are $(-4,87)$ and $(2,-21)$

Classifying the Turning Points

The quickest way to classify most turning points is to use the SECOND DIFFERENTIAL TEST.

Step 1: $\frac{dy}{dx} = 3x^2 + 6x - 24 \Longrightarrow \frac{d^2y}{dx^2} = 6x + 6$

Step 2: Evaluate the second differential at the coordinates of your stationary points and use the following GOLDEN RULE

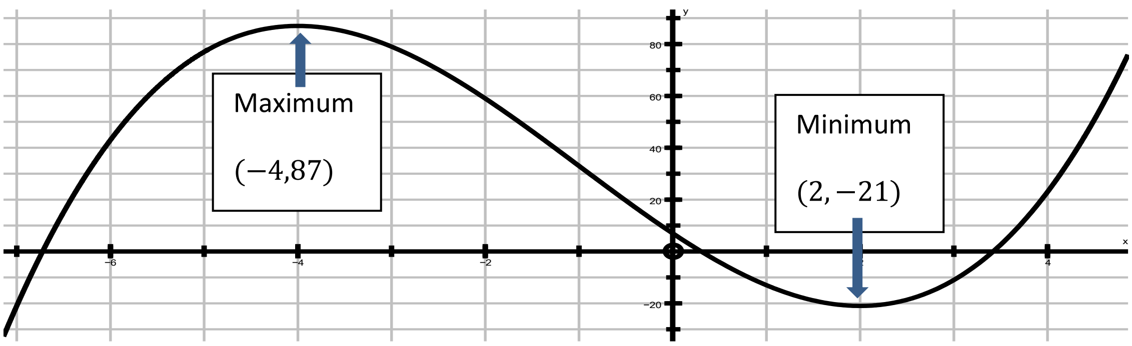
IF $\frac{d^2y}{dx^2} > 0$ then we have a MINIMUM POINT

If $\frac{d^2y}{dx^2} < 0$ then we have a MAXIMUM POINT

At $x = 2$, $\frac{d^2y}{dx^2} = 6x + 6 = 6(2) + 6 = 18 > 0$ Hence a minimum point

At $x = -4$, $\frac{d^2y}{dx^2} = 6x + 6 = 6(-4) + 6 = -18 < 0$ Hence a maximum point

The sketch below indicates this



The Binominal Theorem (Video Link: http://youtu.be/_KfvquVo_KQ)

This first section reviews how we expand expressions of the form $(a+b)^n$ quickly and efficiently

<table>
<tr>
<td>Pascal's triangle allows us to expand expressions of the form $(1+x)^n$ very easily

Pascal's Triangle begins with:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1</td>
<td>This triangle tells us that:

$(1+x)^0 = 1$
$(1+x)^1 = 1 + 1x$
$(1+x)^2 = 1 + 2x + 1x^2$
$(1+x)^3 = 1 + 3x + 3x^2 + 1x^3$
$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + 1x^4$

Observe how the coefficients are the numbers from Pascal's triangle

Can you work out what $(1+x)^5$ would be?</td>
</tr>
</table>

However, we can use this method to expand more general expressions, such as $(2-3x)^4$

We know that: $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + 1x^4$

More generally: $(a+b)^4 = 1a^4 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + 1b^4$

So to find $(2-3x)^4$ we let $a = 2$ and $b = -3x$

$$(2-3x)^4 = 1(2)^4 + 4(2)^3(-3x)^1 + 6(2)^2(-3x)^2 + 4(2)(-3x)^3 + 1(-3x)^4$$

[Now work out each of the brackets]

$$= (16) + 4(8)(-3x) + 6(4)(9x^2) + 4(2)(-27x^3) + (81x^4)$$

$$= 16 + 32(-3x) + 24(9x^2) + 8(-27x^3) + 81x^4$$

$$= 16 - 96x + 216x^2 - 216x^3 + 81x^4$$

Try these: a) $(1+2x)^3$ b) $(2-x)^4$

We can also use this method to expand more complex looking expressions

(Video Link: http://youtu.be/rh-8WCHb7oY)

We know that $(a+b)^3 = 1a^3 + 3a^2b + 3a^1b^2 + 1b^3$

So, $$\left(2x - \frac{1}{x}\right)^3 = (2x)^3 + 3(2x)^2\left(-\frac{1}{x}\right)^1 + 3(2x)^1\left(-\frac{1}{x}\right)^2 + \left(-\frac{1}{x}\right)^3$$

$$= (8x^3) + 3(4x^2)\left(-\frac{1}{x}\right) + 3(2x)\left(\frac{1}{x^2}\right) + \left(-\frac{1}{x^3}\right)$$

$$= 8x^3 + 12x^2\left(-\frac{1}{x}\right) + 6x\left(\frac{1}{x^2}\right) - \frac{1}{x^3}$$

$$= 8x^3 - 12x + \frac{6}{x} - \frac{1}{x^3}$$

Try these $\quad a)\ \left(2+\frac{1}{x}\right)^3 \quad b)\ \left(3x-\frac{1}{x}\right)^3$

Finding Individual Terms (Video Link: http://youtu.be/dRkQ9nRk8rU)

We can also find ONE TERM in the expansion WITHOUT HAVING to do the entire binomial expansion

Generally: To find a coefficient of x^r in the expansion of $(a+bx)^n$ we need to find $nC_r(a)^{n-r}(bx)^r$

Where $nC_r = \frac{n!}{r!(n-r)!}$

Example 1: Find the coefficient of x^2 in the expansion of $(2-3x)^{10}$

$$10C_2(2)^{10-2}(-3x)^2 = \frac{10!}{2!\,8!}(256)(9x^2) = 45(256)(9x^2) = 103680x^2$$

$$\left[Remember\ that: \frac{10!}{2!\,8!} = \frac{10\times 9\times 8\times 7\times \ldots 2\times 1}{2\times 1\times 8\times 7\times 6\times \ldots 1} = 5\times 9 = 45\right]$$

Example 2: Find the constant term in the expansion of $\left(2x-\frac{1}{x}\right)^6$

The constant term is the term where the x cancels out leaving just a number.

$$6C_3(2x)^3\left(-\frac{1}{x}\right)^3 = 30(8x^3)\left(-\frac{1}{x^3}\right) = 160x^3\left(-\frac{1}{x^3}\right) = -\frac{160x^3}{x^3} = -160$$

Mock Questions

1) Expand $(2+x)^5$

2) Solve the inequality $x^2+7x-18>0$

3) Find the values of x for which $2x^3+3x^2-72x+1$ is an increasing function

4) Find the equation of the line passing through $(2,-3)$ with a gradient of 3

5) Find the equation of the perpendicular bisector of $(5,1)$ and $(8,9)$

6) Find the centre and radius of the circle given by the equation:
$$x^2+8x+y^2-4y=2$$

7) Find the equation of the tangent of the following circle at the point $(5,1)$
$$(x+3)^2+(y-2)^2=65$$

8) Show that $x=1$ is a root of the equation $x^3+7x^2+2x-10=0$
Hence, fully factorise and find (if possible) the other two roots

9) Sketch $y=(x+4)(x-3)(x+2)$

10) Solve: $\sin^2 x - sinx = \cos^2 x$

11) Find the equation of the tangent to the curve $y=3x^2-2x$ at the point $x=2$

12) Find and classify the turning points for the function: $y=2x^3+15x^2+36x-2$

13) Differentiate the following

a) $y=x^4-x^3$
b) $y=\sqrt{x}-\frac{5}{x^2}$
c) $y=\frac{3}{\sqrt{x}}+x^2$